

GRAVITINO ZERO MODES ON $U(1)_R$ STRINGS

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Abstract

We consider theories with a spontaneously broken *gauged* R-symmetry, which can only occur in supergravity models. These give rise to cosmic R-strings upon which gravitino zero modes can exist. We construct solutions to the Rarita-Schwinger spin-3/2 equation describing the gravitino in the field of these cosmic strings and show that under some conditions these solutions *may* give rise to gravitino currents on the string. We discuss further mathematical and physical questions associated with these solutions.

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I. INTRODUCTION

Supersymmetric models of particle physics interactions typically contain operators that cause rapid proton decay which must be removed somehow if the theory is to be phenomenologically consistent. One way of doing this is to use R-parity [1] as a symmetry of the Lagrangian. R-parity, if unbroken, will also prevent the lightest supersymmetric particle (LSP) from decaying, thus providing a candidate for WIMP dark matter [2].

There are some potential problems with the use of R-parity, however. Though no consistent theory of quantum gravity is yet known to exist, various conjectures concerning “low energy” effects of gravitational fluctuations have been made. In particular, the possibility that so-called wormhole configurations might exist and have to be summed over in the Euclidean path integral formulation of quantum gravity was at one point thought to be able to provide a solution to the cosmological constant problem [3]. While this possibility was not realized [4], it became clear that if wormhole configurations did in fact have to be included in the functional integral for gravity, then *global* symmetries would be in danger of being explicitly broken by these effects. Essentially, global charge can “pass” through a wormhole, going out of our universe. The sum over wormhole configurations then induces an effective field theory in our universe that will necessarily include operators that break the global symmetry explicitly.

An examination of these effects for R-parity [5] as well as for other important global symmetries such as the Peccei-Quinn symmetry of axion models [6,7] shows that wormholes can deliver devastating blows to theories requiring such symmetries.

Wormholes can only destabilize *global* symmetries. Thus a natural solution to the problem of how to keep R-parity as a good symmetry would be to gauge the full $U(1)_R$ symmetry of which R-parity is a Z_2 subgroup. This can only be done, however, in a *locally* supersymmetric theory due to the commutation relations between the R generators and the supersymmetry generators; gauged R symmetry requires gauged supersymmetry [8].

Recently, such gauged $U(1)_R$ models have been constructed for phenomenological purposes [9,10]. The $U(1)_R$ must be spontaneously broken for consistency and as is well known, the breaking of a $U(1)$ symmetry gives rise to cosmic string configurations [11]. What is interesting about gauged R symmetry is that the gravitino must *necessarily* carry a non-zero R charge. This then allows for the possibility that gravitino zero modes may form on the string. This is what we investigate in this work.

Fermion zero modes in topologically non-trivial backgrounds have been studied extensively in the past, providing results that are interesting both from the mathematical and physical points of view. The Dirac equation has been analysed in various dimensions using kinks, vortices, 'tHooft-Polyakov monopoles and instantons as background fields [12]. These studies have not only yielded explicit solutions, but have also led to index theorems and to connections with the axial anomaly.

Jackiw and Rossi [13] looked at Dirac fermions coupled to a charged scalar in the background of an Abrikosov vortex in the 2-D Abelian-Higgs model. An index theorem for the above configuration was found by E. Weinberg [14] and subsequently Witten [15] studied the Dirac fermion in a cosmic string background in $(3+1)$ -dimensions and found solutions corresponding to transverse fermion zero modes trapped in the string, that gave rise to superconducting currents. These latter results may have strong astrophysical implications,

insofar as the observational evidence for cosmic strings is concerned [16].

In this article we will study spin-3/2 fermions in the background of a cosmic R string in $(3+1)$ dimensions, using the Rarita-Schwinger equation. We look for solutions to the gravitino field equations in such a string background, considering two separate cases: (i) when $U(1)_R$ is broken but supergravity is unbroken; (ii) both $U(1)_R$ and supergravity are broken. In the case where supergravity is unbroken (i.e. the gravitino is massless) we find solutions corresponding to fermions trapped on the string, with a non-zero fermionic current along the string. We speculate that these solutions might also permit superconducting currents. In the supergravity broken phase, we find solutions that are localized on the string, and carry zero R-current. It must, however, be emphasized, that we have found only *some* solutions for the system considered; others may exist. We also present toy models of supergravity where the $U(1)_R$ symmetry is spontaneously broken, while SUGRA (supergravity) is broken in one case and left as an exact symmetry in another.

Section II is devoted to a brief review of R-symmetry and $N = 1$ supergravity, and the subsequent construction of 2 toy models. In Section III we obtain the solutions to the gravitino field equations. Finally, we present our conclusions and possible new lines of study in Section IV.

II. R-INVARIANCE AND SUPERSYMMETRY

We first present a brief review of R -symmetry in this section and then proceed to construct simple toy models which incorporate this symmetry in a consistent fashion. In global SUSY the R -transformations are defined in terms of superfields as,

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) &\rightarrow e^{-in_i\alpha}\Phi(x, e^{-i\alpha}\theta, e^{i\alpha}\bar{\theta}), \\ V(x, \theta, \bar{\theta}) &\rightarrow V(x, e^{-i\alpha}\theta, e^{i\alpha}\bar{\theta}),\end{aligned}\tag{2.1}$$

where $\theta, \bar{\theta}$ are the Grassmann coordinates of superspace, Φ is a chiral superfield (scalar multiplet) with R -charge n_i and V is a gauge vector multiplet. In addition, if we want the action to be R -invariant, we must require that the superpotential g have R -charge = 2, i.e.

$$g(\Phi_i) \rightarrow e^{-2i\alpha}g(\Phi_i)\tag{2.2}$$

In terms of the component fields, the R -transformation behaves like an axial $U(1)$ transformation when acting on fermions. In particular, fermions belonging to a scalar multiplet with charge n_i will have an axial charge = $n_i - 1$, while all gauginos belonging to the gauge vector multiplets will have an axial R -charge 1. The scalar fields belonging to the multiplet Φ_i have charge n_i . The above is true in general, for both global and local R -transformations.

There is a subtlety associated with local R -symmetry, however. If we consider the commutation relations between the R symmetry generator and the supersymmetry generators we have [17]:

$$[Q_\alpha, R] = i(\gamma_5)_\alpha{}^\beta Q_\beta.\tag{2.3}$$

This implies that if the R-symmetry is gauged, this relation can only be maintained if the Q_α are gauged as well, i.e. in a *locally* supersymmetric theory [8]. The important point to

note is that, in the locally R -symmetric theory, the gravitino and the R -photino both have axial R -charges 1:

$$\begin{aligned} e_m^\mu &\rightarrow e_m^\mu \text{ (graviton/vielbien fields)} \\ \psi_\mu &\rightarrow \exp(-i\gamma_5\alpha)\psi_\mu \text{ (gravitino)} \\ \lambda^R &\rightarrow \exp(-i\gamma_5\alpha)\lambda^R \text{ (R-photino)} \end{aligned} \quad (2.4)$$

A. $N = 1$ Supergravity

An $N = 1$ supergravity Lagrangian with canonical kinetic terms for the Yang-Mills fields, is specified by the Kähler potential, $G(\phi, \phi^*)$, which is a real, gauge-invariant function of the scalar fields ϕ_i . For our purposes, we will consider simple toy models, where the matter multiplets are coupled *only* to the $U(1)_R$ gauge fields. In such models, the scalar potential is given by,

$$-e^{-1}\mathcal{L}_P = \frac{1}{\kappa^4}e^G[G^i(G^{-1})^j_i G_j - 3] + \frac{1}{2\kappa^4}D^2 \quad (2.5)$$

where

$$\begin{aligned} G^i &= \partial G / \partial \phi_i, \\ G_i &= \partial G / \partial \phi^{*i}, \\ G^i_j &= \partial^2 G / \partial \phi_i \partial \phi^{*j}, \\ \text{and } D &= eG^i n_i \phi_i. \end{aligned}$$

In addition, the function G must be of the form,

$$G(\phi, \phi^*) = J(\phi, \phi^*) + \ln |\kappa^3 g(\phi)|^2 \quad (2.6)$$

where J is referred to as the Kähler metric and g is the superpotential which is an analytic (chiral) function of the fields. In this formulation of supergravity [18] the gravitinos and gauginos have “naive” R -charge = 0. However, to obtain the physical couplings one has to perform the reverse chiral rotations on the gravitino field, as pointed out in [9],

$$\psi_{\mu L} \rightarrow \left(\frac{g^*}{g}\right)^{1/4} \psi_{\mu L} \quad \psi_{\mu R} \rightarrow \left(\frac{g}{g^*}\right)^{1/4} \psi_{\mu R}. \quad (2.7)$$

These rotations ensure that the gravitino has appropriate couplings to the R -photon, and the gravitino sector of the Lagrangian is then given by,

$$\begin{aligned} \mathcal{L} &= -i\bar{\psi}_{\mu L}\gamma^{\mu\nu\lambda}[\partial_\nu - ieA_\nu]\psi_{\lambda L} - i\bar{\psi}_{\mu R}\gamma^{\mu\nu\lambda}[\partial_\nu + ieA_\nu]\psi_{\lambda R} \\ &+ \frac{1}{\kappa}e^{G/2}\left(\frac{g}{g^*}\right)^{1/2}\bar{\psi}_{\mu L}\gamma^{\mu\nu}\psi_{\nu R} + \frac{1}{\kappa}e^{G/2}\left(\frac{g^*}{g}\right)^{1/2}\bar{\psi}_{\mu R}\gamma^{\mu\nu}\psi_{\nu L}. \end{aligned} \quad (2.8)$$

The full $N = 1$ lagrangian with Yang-Mills couplings to matter has a much more complicated form and can be found in [18]. However, we are only interested in *classical* solutions to

the field equations obtained from the supergravity lagrangian, and hence we restrict ourselves only to a particular set of classical configurations for which the gaugino and other spin-1/2 fields in the theory vanish identically. Therefore, quadratic terms which generate gravitino-gaugino and gravitino-fermionic mixings can be ignored by simply setting the gaugino and other fermionic fields to zero. Furthermore, higher order, four-fermi type gravitino vertices are suppressed by inverse powers of the Planck scale and hence are negligible.

Now any consistent gauge theory with axial vector couplings must be anomaly free. Thus, if we set up a supergravity model with $U(1)_R$ symmetry, we must ensure that the charges of the matter multiplets in the theory satisfy appropriate anomaly cancellation conditions. Moreover, the superpotential must have R -charge 2, and so every term in the superpotential must satisfy $\sum n_i = 2$. The vanishing of the anomalous triangle diagram requires,

$$Tr R^3 = 0. \quad (2.9)$$

Since gravity couples to all fields with equal strength, the mixed gravitational anomaly can be removed by demanding,

$$Tr R = 0 \quad (2.10)$$

where the trace is over all the fermions in the theory.

B. A Toy Model

1. Case I: SUGRA and $U(1)_R$ Broken

With all the previously mentioned ideas in mind let us try to construct a toy model where we can consistently implement the R -symmetry and break it spontaneously along with supergravity. For simplicity we assume that $U(1)_R$ is the only gauge symmetry in the theory. Let there be a set of chiral multiplets $\{\Phi_0, \Phi_i\}$ ($i = 1, 2, \dots, k$) with R -charges $\{n_0, n_i\}$. We propose a non-canonical kinetic term for the scalar field ϕ_0 belonging to the multiplet Φ_0 . All other fields are assumed to have canonical terms, so that the Kähler potential has the following form,

$$G(\phi, \phi^*) = -\frac{3}{2} \ln[(\phi_0 + \phi_0^*)^2 \kappa^2] + \sum \phi_i \phi_i^* \kappa^2 + \ln[\kappa^6 |g(\phi_i)|^2] \quad (2.11)$$

$$g(\phi_i) = m^2(\phi_1 - \frac{\sqrt{2}\kappa^2}{3}\phi_1^2\phi_2) + \sum_i \lambda_i \prod_j \phi_j^{L_{ij}} \quad (2.12)$$

$$L_{ij}n_j = 2 \quad (2.13)$$

$$m \sim \mathcal{O}(1/\kappa). \quad (2.13)$$

Since the Kähler potential is gauge invariant, the above form implies that ϕ_0 must be neutral i.e. $n_0 = 0$. The choice of the superpotential (2.12) fixes the R -charges of the fields ϕ_1 and ϕ_2 so that $n_1 = 2$ and $n_2 = -2$. The choice of the non-canonical term was of course, made arbitrarily. The only purpose of that term is to make our calculations simple, by reducing the scalar potential into a sum of squares whose minimum value is at zero. This ensures that the cosmological constant vanishes. The scalar potential is then given by

$$\begin{aligned}
V(\phi, \phi^*) &= \frac{1}{\kappa^4} e^{\kappa^2 \Sigma |\phi_i|^2} \frac{|g|^2 \kappa}{|\phi_0 + \phi_0^*|^3} \left[\sum_{i=1}^k \left| \kappa^2 \phi_i^* + \frac{g^i}{g} \right|^2 \right] \\
&+ \frac{1}{2\kappa^4} e^2 \left(\sum_{i=1}^k n_i \phi_i \left[\kappa^2 \phi_i^* + \frac{g^i}{g} \right] \right)^2.
\end{aligned} \tag{2.14}$$

The charges of three of the multiplets were fixed uniquely by the form of the superpotential. Additional multiplets with non-trivial R -charges will, however, be required for purposes of anomaly cancellation. We denote the charges of the fermions in the multiplets by $f_i \equiv n_i - 1$. Therefore, $f_0 = -1$, $f_1 = 1$ and $f_2 = -3$. The mixed gravitational anomaly and the triangle anomaly are removed if,

$$\begin{aligned}
Tr R &= \sum_{i=3}^k f_i - 1 + 1 - 3 + 1 - 21 = \sum_{i=3}^k f_i - 23 = 0 \\
Tr R^3 &= \sum_{i=3}^k f_i^3 + (-1)^3 + 1^3 + (-3)^3 + 1^3 + 3 \times 1^3 = \sum_{i=3}^k f_i^3 - 23 = 0.
\end{aligned} \tag{2.15}$$

The gravitino contribution to the mixed anomaly is -21 times, while the contribution to the triangle anomaly is 3 times that of the gaugino [9], [19]. The simplest solution is to have $k = 25$ i.e. introduce 23 additional scalar multiplets each with R -charge = 2 into the superpotential in such a way that (2.12) is always satisfied. Thus, we can build a superpotential that fixes the charges of the fields in such a way that the anomaly cancellation conditions are satisfied:

$$g(\phi_i) = m^2(\phi_1 - \frac{\sqrt{2}\kappa^2}{3}\phi_1^2\phi_2) + \sum_{i=3}^{25} \lambda_i \phi_i^2 \phi_2. \tag{2.16}$$

There are other, more economical possibilities, but this one will suffice to make our point. The field configuration that minimises the scalar potential is,

$$|\langle \phi_1 \rangle| = \frac{1}{\kappa}, \quad |\langle \phi_2 \rangle| = \frac{\sqrt{2}}{\kappa}, \quad \langle \phi_{3,4,\dots} \rangle = 0, \tag{2.17}$$

with ϕ_0 left undetermined. In the vacuum sector supergravity is also broken, since the gravitino acquires a mass,

$$m_{3/2} = \frac{1}{\kappa} e^{G/2} = \frac{e^{3/2} m^2}{3|\phi_0 + \phi_0^*|^{3/2} \kappa^{1/2}}. \tag{2.18}$$

Since we have broken a gauged $U(1)_R$ symmetry, vortex configurations will be generated. The winding number n vortex configuration is characterized by the following behaviour of the fields :

$$\phi_1(\vec{r}) = e^{in\theta} f_1(r) \tag{2.19}$$

$$\phi_2(\vec{r}) = e^{-in\theta} f_2(r)$$

$$\begin{aligned}
f_{1,2}(r) &\longrightarrow f_{1,2}^0 r^{|n|}, \quad \text{as } r \rightarrow 0 \\
f_1(r) &\longrightarrow \frac{1}{\kappa}, \quad \text{as } r \rightarrow \infty \\
f_2(r) &\longrightarrow \frac{\sqrt{2}}{\kappa}, \quad \text{as } r \rightarrow \infty
\end{aligned}$$

We use the function \mathcal{F} to denote the generalized interactions of the gravitino with the scalar sector of the theory (see eq. (2.8)), which gives rise to the mass term for the gravitino in the vacuum sector:

$$\begin{aligned}
\mathcal{F}(\phi, \phi^*) &= \frac{1}{\kappa} e^{G/2} \left(\frac{g}{g^*} \right)^{1/2} \\
&\longrightarrow f_0 \frac{\kappa^{1/2} m^2 r^{|n|} e^{in\theta}}{|\phi_0 + \phi_0^*|^{3/2}} \quad \text{as } r \rightarrow 0 \\
&\longrightarrow \frac{e^{3/2} m^2 e^{in\theta}}{3 |\phi_0 + \phi_0^*|^{3/2} \kappa^{1/2}} \quad \text{as } r \rightarrow \infty.
\end{aligned} \tag{2.20}$$

2. Case II: $U(1)_R$ Broken and SUGRA Unbroken

We will now provide another simple example where the R -symmetry is broken spontaneously due to the fact that one of the fields gets a vacuum expectation value and gives rise to a string configuration. However, in this example supergravity will remain as an exact symmetry, and thus the gravitino will remain massless. We assume that the theory has 3 gauge singlets $\{S_1, S_2, S_3\}$ and k other superfields $\{\Phi_i\}$ ($i = 1, 2, \dots, k$). The kinetic terms for all the fields are taken to be canonical. The Kähler potential and the superpotential are chosen to be:

$$G(\phi, \phi^*) = \sum s_i s_i^* \kappa^2 + \sum \phi_i \phi_i^* \kappa^2 + \ln[\kappa^6 |g(\phi_i)|^2] \tag{2.21}$$

$$\begin{aligned}
g &= s_1 s_2 s_3 \left[m^2 (\phi_1 - \frac{\sqrt{2} \kappa^2}{3} \phi_1^2 \phi_2) + \sum_{i=3}^k \lambda_i \phi_i^2 \phi_2 \right] \\
m &\sim \mathcal{O}(1/\kappa).
\end{aligned} \tag{2.22}$$

The superpotential has the same form as in the previous example except for the overall multiplication by the singlet fields. Thus the charges of the scalar fields ϕ_1 and ϕ_2 are 2 and -2 respectively, while all the remaining fields ϕ_3, ϕ_4, \dots have charge = 2. The scalar potential obtained from this particular choice of the superpotential, is

$$\begin{aligned}
V &= e^{\kappa^2 \Sigma(|\phi_i|^2 + |s_i|^2)} |g|^2 \left[\sum_{i=1}^k \left| \kappa^2 \phi_i^* + \frac{g^i}{g} \right|^2 + \sum_{i=1}^3 \left| \kappa^2 s_i^* + \frac{1}{s_i} \right|^2 - 3\kappa^2 \right] \\
&+ \frac{1}{2\kappa^4} e^2 \left(\sum_{i=1}^k n_i \phi_i \left[\kappa^2 \phi_i^* + \frac{g^i}{g} \right] \right)^2.
\end{aligned} \tag{2.23}$$

It is easily seen that this scalar potential is always positive, and its tree level minimum value is at zero. A field configuration that minimizes the potential is given by

$$|\langle\phi_1\rangle| = \frac{1}{\kappa}, \quad |\langle\phi_2\rangle| = \frac{\sqrt{2}}{\kappa}, \quad \langle\phi_{3,4,\dots}\rangle = \langle s_{1,2,3}\rangle = 0. \quad (2.24)$$

The anomaly cancellation conditions require

$$\begin{aligned} TrR &= \sum_{i=3}^k f_i - 1 \times 3 + 1 - 3 + 1 - 21 = \sum_{i=3}^k f_i - 25 = 0 \\ TrR^3 &= \sum_{i=3}^k f_i^3 + 3(-1)^3 + 1^3 + (-3)^3 + 1^3 + 3 \times 1^3 = \sum_{i=3}^k f_i^3 - 25 = 0, \end{aligned} \quad (2.25)$$

where $f_i = n_i - 1 = 1$, represents the R -charges of the fermions, as before. But now we need $k = 27$ to cancel all the anomalies. The field configurations characterizing the vortex are exactly the same as in eq. (2.19). However the mass term for the gravitino is exactly zero:

$$\mathcal{F}(\phi, \phi^*) = \frac{1}{\kappa} e^{G/2} \left(\frac{g}{g^*} \right)^{1/2} = 0. \quad (2.26)$$

Thus we see that models can be constructed which incorporate an anomaly free gauged R -symmetry which is spontaneously broken and that the supersymmetry can either be broken or unbroken at the same time.

III. RARITA-SCHWINGER EQUATIONS IN THE STRING BACKGROUND

We now turn to the question of finding solutions to the gravitino equation of motion in the background of a vortex configuration for the R gauge field. This entails looking at the Rarita-Schwinger equation coupled to the R -photon. This equation takes the form:

$$\begin{aligned} \mathcal{L} &= -i\bar{\psi}_{\mu L} \gamma^{\mu\nu\lambda} [\partial_\nu - ieA_\nu] \psi_{\lambda L} - i\bar{\psi}_{\mu R} \gamma^{\mu\nu\lambda} [\partial_\nu + ieA_\nu] \psi_{\lambda R} \\ &+ \mathcal{F}(\phi, \phi^*) \bar{\psi}_{\mu L} \gamma^{\mu\nu} \psi_{\nu R} + \mathcal{F}^*(\phi^*, \phi) \bar{\psi}_{\mu R} \gamma^{\mu\nu} \psi_{\nu L} \end{aligned} \quad (3.1)$$

We have introduced the coupling of the gravitino to the scalars through the arbitrary complex function of the scalar fields, $\mathcal{F}(\phi, \phi^*)$, which was defined in eq. (2.20). However, since the Lagrangian has a $U(1)_R$ symmetry, \mathcal{F} must have R -charge 2. Thus the Lagrangian is invariant under,

$$\begin{aligned} \psi_\mu &\rightarrow \exp(-ie\gamma_5 \Lambda) \psi_\mu \\ A_\mu &\rightarrow A_\mu - \partial_\mu \Lambda \\ \mathcal{F} &\rightarrow \exp(-2ie\Lambda) \mathcal{F}. \end{aligned} \quad (3.2)$$

We now suppose that the R -photon and the scalar fields take on the appropriate form to represent a vortex solution. We then require that:

$$\mathcal{F} = e^{iK\theta} \mathcal{H}(r), \quad (3.3)$$

$$\begin{aligned} \mathcal{H}(r) &\longrightarrow \mathcal{H}_0 r^{|K|}, & \text{as } r \rightarrow 0 \\ \mathcal{H}(r) &\longrightarrow \mathcal{H}_\infty, & \text{as } r \rightarrow \infty \end{aligned}$$

where $\mathcal{H}(r)$ is a real function, $\mathcal{H}_0, \mathcal{H}_\infty > 0$ are constants and K must be an integer to ensure that the interaction terms are single-valued. The integer K depends not only on the vortex winding number n , but also on the exact form of the function \mathcal{F} . If we assume that the string configuration has cylindrical symmetry about the z -axis, the gauge field for the vortex solution, will only have an azimuthal component:

$$\begin{aligned} \vec{A}(\vec{r}) &= A(r) \hat{e}_\theta. \\ A(r) &\longrightarrow 0 & \text{as } r \rightarrow 0 \\ A(r) &\longrightarrow \frac{n}{qr} & \text{as } r \rightarrow \infty \end{aligned} \quad (3.4)$$

where q is the charge of the scalar field that gets a VEV. The gauge field goes to zero at the origin and at infinity, and carries a net topological flux $n/2q$. From (3.1) we obtain the Rarita-Schwinger equations for the spinor fields,

$$i\gamma^{\mu\nu\lambda}[\partial_\nu - ieA_\nu]\psi_{\lambda L} - \mathcal{F}(\phi, \phi^*)\gamma^{\mu\nu}\psi_{\nu R} = 0, \quad (3.5)$$

$$i\gamma^{\mu\nu\lambda}[\partial_\nu + ieA_\nu]\psi_{\lambda R} - \mathcal{F}^*(\phi^*, \phi)\gamma^{\mu\nu}\psi_{\nu L} = 0.$$

In the vacuum sector, the fermions get a mass \mathcal{H}_∞ , while in the string core, they are massless. We will solve eq.(3.5) with a background gauge field given by eq.(3.4) in the next section.

A. Solutions to the Rarita-Schwinger equations

Case I (SUGRA unbroken):

In this case, the gravitino obeys the massless Rarita-Schwinger equations,

$$i\gamma^{\mu\nu\lambda}[\partial_\nu - ieA_\nu]\psi_{\lambda L} = 0, \quad (3.6)$$

$$i\gamma^{\mu\nu\lambda}[\partial_\nu + ieA_\nu]\psi_{\lambda R} = 0.$$

These equations can be rewritten in terms of the gauge covariant derivatives as

$$\begin{aligned} &\gamma^{\mu\nu\lambda}D_\nu\psi_{\lambda L} \\ &= \gamma^\nu D_\nu\psi_L^\mu + \gamma^\mu\gamma^{\nu\lambda}D_\nu\psi_{\lambda L} - D^\mu(\gamma \cdot \psi_L) = 0 \end{aligned} \quad (3.7)$$

One can get the right-handed field equations by replacing the covariant derivatives D_i with D_i^* . Contracting this equation with γ_μ yields

$$\begin{aligned}\gamma^{\nu\lambda}D_\nu\psi_{\lambda L} &= 0 \\ \gamma^\nu D_\nu\psi_L^\mu &= D^\mu(\gamma \cdot \psi_L).\end{aligned}\tag{3.8}$$

Now, acting on eq. (3.7) with the operator D_μ , we get,

$$\gamma^{\mu\nu\lambda}D_\mu D_\nu\psi_{\lambda L} = -\frac{ie}{2}\gamma^{\mu\nu\lambda}F_{\mu\nu}\psi_{\lambda L} = 0.\tag{3.9}$$

But the vector potential has only an azimuthal component which means that the only non-zero components of $F_{\mu\nu}$ are F_{12} and F_{21} . Therefore, we find,

$$\gamma^0\psi_{0L,R} + \gamma^3\psi_{3L,R} = 0.\tag{3.10}$$

If we impose the additional condition that $\gamma \cdot \psi_{L,R} = 0$ i.e. $\gamma^1\psi_{1L,R} + \gamma^2\psi_{2L,R} = 0$, we find that each of the 4 components of the spinor satisfies the Dirac equation

$$\gamma^\nu D_\nu\psi_{L,R}^\mu = 0.\tag{3.11}$$

Now we can exploit the symmetry of the problem and assume that the solutions are separable i.e. $\psi_{\mu L,R} = \beta(z, t)\tilde{\psi}_{\mu L,R}(x, y)$. One can easily see that the spinors must then satisfy the transverse Dirac equation,

$$(\gamma^1 D_1 + \gamma^2 D_2)\psi_{\mu L} = 0,\tag{3.12}$$

$$(\gamma^1 D_1^* + \gamma^2 D_2^*)\psi_{\mu R} = 0.$$

In addition, the left-handed fields have to satisfy

$$(\partial_0 + \sigma_3\partial_3)\psi_{\mu L} = 0,\tag{3.13}$$

while the right-handed fields satisfy

$$(\partial_0 - \sigma_3\partial_3)\psi_{\mu R} = 0.\tag{3.14}$$

We now have the option of choosing the fields to be eigenstates of σ_3 . A left-handed spin-up (down) state ends up being a right (left)-mover, while a right-handed spin-up (down) state is a left (right)-mover. We will focus only on the left-handed spin-up fields – the other cases can be handled in a similar fashion. Thus we take:

$$\psi_{\mu L} = \alpha(z - t) \begin{pmatrix} u(r, \theta) \\ 0 \end{pmatrix}.$$

Since the spinors are zero modes of the transverse Dirac operator (3.12), the function $u(r, \theta)$ is a solution to the differential equation:

$$[\partial_r + eA(r) + \frac{i}{r}\partial_\theta]u(r, \theta) = 0,\tag{3.15}$$

which implies that

$$u(r, \theta) = \sum_m a_m r^m e^{im\theta} \exp(-e \int^r A(r) dr), \quad (3.16)$$

$$m = 0, 1, 2, \dots, \left[\frac{ne}{q} - 1\right].$$

where $[N]$ denotes the greatest integer smaller than N . The right-moving solutions are therefore given by

$$\begin{aligned} \psi_{1L} &= -i\psi_{2L} = -i\alpha(z-t) \begin{pmatrix} u(r, \theta) \\ 0 \end{pmatrix}, \\ \psi_{0L} &= -\psi_{3L} = -\beta(z-t) \begin{pmatrix} u_1(r, \theta) \\ 0 \end{pmatrix}, \\ \psi_{1R} &= i\psi_{2R} = i\alpha_1(z-t) \begin{pmatrix} 0 \\ v(r, \theta) \end{pmatrix}, \\ \psi_{0R} &= -\psi_{3R} = -\beta_1(z-t) \begin{pmatrix} 0 \\ v_1(r, \theta) \end{pmatrix}. \end{aligned} \quad (3.17)$$

Here u, u_1 are solutions to (3.15), while v, v_1 are solutions to

$$[\partial_r + eA(r) - \frac{i}{r}\partial_\theta]v = 0 \quad (3.18)$$

It is not difficult to verify that (3.17) satisfy the covariant condition $D \cdot \psi = 0$. The solutions (3.16) are normalizable provided $ne/q > 1$ since asymptotically far from the string, $A(r) = n/qr$ and $u \sim r^{-ne/q}$.

The left-handed fermionic current along the z -axis is given by $= -e\bar{\psi}_{\mu L}\gamma^{\mu 3\lambda}\psi_{\lambda L}$ and for the spin-up solutions (3.17) is given by $e|\alpha(z-t)|^2|u(r, \theta)|^2$. In the absence of a clear proof, we speculate that these non-zero currents could lead to superconducting/persistent R -currents in the string.

The calculations so far, have closely paralleled that of reference [20], where the zero modes of the massless Dirac equation were studied in the presence of a magnetic field in two spatial dimensions. We will now look at the case where supersymmetry is broken and the gravitino becomes massive.

Case II (SUGRA broken):

In this scenario, as indicated in eqns.(3.5), the gravitino acquires a mass away from the string core. Consequently, the equations for the left-handed and right-handed fermions are coupled and we have a much more complicated situation than the SUSY unbroken case. However, based on the previous calculation we can hazard a guess as to the form of the solutions – in particular, we expect that there will be solutions which are eigenstates of σ_3 in the presence of the R magnetic field. We thus propose the following ansatz:

$$\psi_{\mu L} = \begin{pmatrix} U_\mu \\ 0 \end{pmatrix} \quad (3.19)$$

$$\psi_{\mu R} = \begin{pmatrix} 0 \\ V_\mu \end{pmatrix}$$

Writing out the field equations in terms of the spinor components we get,

$$(i) \quad D_1 U_2 - D_2 U_1 - 2\mathcal{F}(V_1 - iV_2) = 0 \quad (3.20)$$

$$(ii) \quad (D_2 - iD_1)U_3 - \partial_3(U_2 - iU_1) + 2\mathcal{F}V_3 = 0$$

$$(iii) \quad \partial_3 U_0 - \partial_0 U_3 - 2\mathcal{F}V_2 = 0$$

$$(iv) \quad (\partial_0 + \partial_3)U_2 - D_2(U_0 + U_3) - 2\mathcal{F}(V_0 + V_3) = 0$$

$$(v) \quad \partial_3 U_0 - \partial_0 U_3 - 2i\mathcal{F}V_1 = 0$$

$$(vi) \quad iD_1(U_0 + U_3) - i(\partial_0 + \partial_3)U_1 - 2\mathcal{F}(V_3 + V_0) = 0$$

$$(vii) \quad \partial_0(U_2 - iU_1) - (D_2 - iD_1)U_0 - 2\mathcal{F}V_0 = 0.$$

$$(a) \quad D_1^* V_2 - D_2^* V_1 - 2\mathcal{F}^*(U_1 + iU_2) = 0 \quad (3.21)$$

$$(b) \quad (D_2^* + iD_1^*)V_3 - \partial_3(V_2 + iV_1) + 2\mathcal{F}^*U_3 = 0$$

$$(c) \quad \partial_3 V_0 - \partial_0 V_3 - 2\mathcal{F}^*U_2 = 0$$

$$(d) \quad (\partial_0 + \partial_3)V_2 - D_2^*(V_0 + V_3) - 2\mathcal{F}^*(U_0 + U_3) = 0$$

$$(e) \quad \partial_3 V_0 - \partial_0 V_3 + 2i\mathcal{F}^*U_1 = 0$$

$$(f) \quad D_1^*(V_0 + V_3) - (\partial_0 + \partial_3)V_1 - 2i\mathcal{F}^*(U_3 + U_0) = 0$$

$$(g) \quad \partial_0(V_2 + iV_1) - (D_2^* + iD_1^*)V_0 - 2\mathcal{F}^*U_0 = 0$$

The equations (3.20) (iii), (3.20) (v), (3.21) (c) and (3.21) (e) imply that $U_1 = iU_2$ and $V_1 = -iV_2$. Using these relations and subtracting eq.(vii) from eq.(ii) we get,

$$(D_2 - iD_1)(U_3 + U_0) - 2(\partial_3 + \partial_0)U_2 + 2\mathcal{F}(V_0 + V_3) = 0. \quad (3.22)$$

On the other hand eqns.(iv), (vi) yield

$$(D_2 - iD_1)(U_3 + U_0) - 2(\partial_3 + \partial_0)U_2 + 4\mathcal{F}(V_0 + V_3) = 0. \quad (3.23)$$

We therefore find that $V_0 = -V_3$. A similar manipulation of eqns. (3.21) (b, d, f, g) shows that $U_0 = -U_3$. This, then implies that

$$\begin{aligned} (\partial_0 + \partial_3)U_2 &= 0, \\ (\partial_0 + \partial_3)V_2 &= 0, \end{aligned} \quad (3.24)$$

i.e. $U_2 = U_2(x, y, z - t)$ and $V_2 = V_2(x, y, z - t)$. Let us now define the light-cone variables $X_+ = (z + t)/2$ and $X_- = (z - t)/2$. In terms of these variables eq. (3.20) (iii) can be written as

$$\partial_+ U_0 = \mathcal{F}V_2(X_-). \quad (3.25)$$

The solution to this equation is then

$$U_0(X_+, X_-) = \mathcal{F}V_2(X_-)X_+ + g(X_-) \quad (3.26)$$

where g is an arbitrary function of X_- . But this form of U_0 is in general unbounded in z and t because of the linear dependence on X_+ , unless V_2 vanishes identically. Using a similar argument, one can show that U_2 must also be zero. These results can be shown to hold for the spin-down fields too, except that they happen to be left-movers. Thus the solutions can be characterised by the following set of conditions:

$$\begin{aligned} \psi_{1L,R} &= \psi_{2L,R} = 0, \\ \psi_{0L} &= -\sigma_3\psi_{3L} = \pm\psi_{3L} \\ &= \alpha(z \pm t)\tilde{\psi}_L(x, y) \\ \text{and} \\ \psi_{0R} &= \sigma_3\psi_{3R} = \pm\psi_{3R} \\ &= \alpha(z \pm t)\tilde{\psi}_R(x, y). \end{aligned} \quad (3.27)$$

$$\begin{aligned} i(\sigma_1 D_1 + \sigma_2 D_2)\tilde{\psi}_L(x, y) - 2\mathcal{F}\tilde{\psi}_R(x, y) &= 0 \\ i(\sigma_1 D_1^* + \sigma_2 D_2^*)\tilde{\psi}_R(x, y) + 2\mathcal{F}^*\tilde{\psi}_L(x, y) &= 0. \end{aligned}$$

Let us first try to obtain the right-moving solutions explicitly, i.e.

$$\psi_{0L} = -\psi_{3L} = \alpha(z - t) \begin{pmatrix} u(x, y) \\ 0 \end{pmatrix} \quad (3.28)$$

and

$$\psi_{0R} = -\psi_{3R} = \alpha(z - t) \begin{pmatrix} 0 \\ -iv(x, y) \end{pmatrix}.$$

This would describe a left-handed spin-up Weyl spinor, and a right-handed spin-down Weyl spinor moving along the positive z axis at the speed of light. Using (3.20), (3.21) and (3.27), we can obtain the equations for the complex functions u and v in polar-coordinates defined on the x - y plane:

$$\partial_r u + \frac{i}{r} \partial_\theta u + eA(r)u + 2e^{i(K-1)\theta} \mathcal{H}v = 0 \quad (3.29)$$

$$\partial_r v - \frac{i}{r} \partial_\theta v + eA(r)v + 2e^{-i(K-1)\theta} \mathcal{H}u = 0 \quad (3.30)$$

Adding (3.29) and the complex conjugate of (3.30) we obtain,

$$\partial_r \delta + \frac{i}{r} \partial_\theta \delta + eA(r)\delta + 2e^{i(K-1)\theta} \mathcal{H}\delta^* = 0 \quad (3.31)$$

where, $\delta = (u + v^*)$. On the other hand, subtracting the complex conjugate of (3.30) from (3.29) gives,

$$\partial_r \eta + \frac{i}{r} \partial_\theta \eta + eA(r)\eta - 2e^{i(K-1)\theta} \mathcal{H}\eta^* = 0 \quad (3.32)$$

where $\eta = u - v^*$. We can now follow the analysis of [13] to obtain the solutions to (3.31) and (3.32). We write η and δ as

$$\begin{aligned} \delta &= e^{-e \int A(r) dr} (\delta_1 e^{im\theta} + \delta_2 e^{i(K-1-m)\theta}) \\ \eta &= e^{-e \int A(r) dr} (\eta_1 e^{im\theta} + \eta_2 e^{i(K-1-m)\theta}) \end{aligned} \quad (3.33)$$

where m is an integer $\neq \frac{K-1}{2}$. We notice that (3.32) can be obtained from (3.31) by the replacement, $\mathcal{H} \rightarrow -\mathcal{H}$. It therefore, suffices to concentrate on the solutions of (3.31). Without any loss of generality, we assume that asymptotically away from the string, $\mathcal{H} \rightarrow \mathcal{H}_\infty > 0$.

Since the differential equations involve complex conjugation, they are non-linear and hence need to be linearized. This can be done by writing them in terms of the real and imaginary parts of the δ 's, namely $\delta_{1,2r}$ and $\delta_{1,2i}$ thus reducing (3.31) to the pair of equations:

$$\begin{aligned} \delta'_{1r(i)} - \frac{m}{r} \delta_{1r(i)} + (-)2\mathcal{H}\delta_{2r(i)} &= 0 \\ \delta'_{2r(i)} - \frac{K-1-m}{r} \delta_{2r(i)} + (-)2\mathcal{H}\delta_{1r(i)} &= 0 \end{aligned} \quad (3.34)$$

where the prime denotes a radial derivative. The corresponding equations for the η variables are

$$\begin{aligned} \eta'_{1r(i)} - \frac{m}{r} \eta_{1r(i)} - (+)2\mathcal{H}\eta_{2r(i)} &= 0 \\ \eta'_{2r(i)} - \frac{K-1-m}{r} \eta_{2r(i)} - (+)2\mathcal{H}\eta_{1r(i)} &= 0. \end{aligned} \quad (3.35)$$

Following the arguments in [13] we define a pair of real normalizable functions (u_m, v_m) which satisfy,

$$\begin{aligned} u'_m - \frac{m}{r} u_m + 2\mathcal{H}v_m &= 0 \\ v'_m - \frac{K-1-m}{r} v_m + 2\mathcal{H}u_m &= 0. \end{aligned} \quad (3.36)$$

The general solutions for $K > 0$ are then given by:

$$\delta_1 = a_m u_m, \quad \delta_2 = a_m^* v_m \eta_1 = b_n u_n, \quad \eta_2 = -b_n^* v_n \quad (3.37)$$

and

$$\delta = e^{-e \int A(r) dr} [a_m u_m(r) e^{im\theta} + a_m^* v_m(r) e^{i(K-1-m)\theta}]$$

$$\eta = e^{-e \int A(r) dr} [b_n u_n(r) e^{in\theta} - b_n^* v_n(r) e^{i(K-1-n)\theta}]$$

$$K-1 \geq m, n \geq 0.$$

The a 's and b 's are constants of integration. The functions u_m, v_m must be well behaved both at infinity and at the origin; this forces η and δ to go to zero, whenever $K < 0$. The final form of the right-moving solutions thus obtained is:

$$\begin{aligned} \psi_{0L} = -\psi_{3L} &= \frac{\alpha(z-t)}{2} \begin{pmatrix} e^{-e \int A(r) dr} [a_m u_m(r) e^{im\theta} + a_m^* v_m(r) e^{i(K-1-m)\theta}] \\ + b_n u_n(r) e^{in\theta} - b_n^* v_n(r) e^{i(K-1-n)\theta} \\ 0 \end{pmatrix} \\ \psi_{1L} = \psi_{2L} &= 0 \end{aligned} \quad (3.38)$$

and

$$\begin{aligned} \psi_{0R} = -\psi_{3R} &= \frac{\alpha(z-t)}{2} \begin{pmatrix} 0 \\ -i e^{-e \int A(r) dr} [a_m^* u_m(r) e^{-im\theta} + a_m v_m(r) e^{-i(K-1-m)\theta}] \\ -b_n^* u_n(r) e^{-in\theta} + b_n v_n(r) e^{-i(K-1-n)\theta} \end{pmatrix} \\ \psi_{1R} = \psi_{2R} &= 0 \end{aligned}$$

The case $m, n = \frac{K-1}{2}$ has to be treated separately, because the ansatz (3.33), now involves only one function. Obviously, this can happen only when K is odd. Modifying the ansatz, we obtain,

$$\begin{aligned} \delta &= e^{-e \int A(r) dr} \delta_1 e^{i \frac{(K-1)}{2} \theta} \\ \eta &= e^{-e \int A(r) dr} \eta_1 e^{i \frac{(K-1)}{2} \theta} \end{aligned} \quad (3.39)$$

The solutions are easy to obtain and fit into the form shown in (3.38). In particular, we see that $u_{K/2-1/2}(r) = v_{K/2-1/2}(r) = r^{K/2-1/2} \exp[-2\int \mathcal{H}(r)dr]$. Therefore, we have seen that the right-moving solutions given by (3.28) exist only when $K > 0$.

What about left-movers? Going through an analysis similar to the above we see that left-movers are allowed only when $K < 0$. Defining $K' = -K > 0$ we obtain the following as the solutions,

$$\begin{aligned} \psi_{0L} = \psi_{3L} &= \frac{\alpha(z+t)}{2} \begin{pmatrix} 0 \\ e^{\int A(r)dr} [c_m^* u_m(r) e^{-im\theta} + c_m^* v_m(r) e^{-i(K'-1-m)\theta} \\ + d_n^* u_n(r) e^{-in\theta} - d_n^* v_n(r) e^{-i(K'-1-n)\theta}] \end{pmatrix} \\ \psi_{1L} = \psi_{2L} &= 0 \end{aligned} \tag{3.40}$$

and

$$\begin{aligned} \psi_{0R} = \psi_{3R} &= \frac{\alpha(z+t)}{2} \begin{pmatrix} -ie^{\int A(r)dr} [c_m^* u_m(r) e^{im\theta} + c_m v_m(r) e^{i(K'-1-m)\theta} \\ -d_n^* u_n(r) e^{in\theta} + d_n v_n(r) e^{i(K'-1-n)\theta}] \\ 0 \end{pmatrix} \\ \psi_{1R} = \psi_{2R} &= 0 \end{aligned}$$

$$K' - 1 \geq m, n \geq 0.$$

Another way to obtain left-movers is to have fermions with R -charge $-e$ and couple them to \mathcal{F}^* . One can easily check that these solutions satisfy the conditions $D \cdot \psi = 0$ and $\gamma \cdot \psi = 0$. However, the charge density and current density for these solutions, given by $-e \bar{\psi}_{\mu L} \gamma^{\mu 3\lambda} \psi_{\lambda L}$ happens to be identically zero. Thus they cannot give rise to superconducting currents of any kind.

There are two points to note about the solutions that we have obtained. First, they are simply the zero modes of the transverse Dirac operator in a vortex background, and the properties of these zero modes are well-known, both from index theorems and from explicit solutions [13], [14]. Second, as one sees in the case of spin-1/2 fermions, the solutions (3.38) are localized along the string. The functions are exponentially suppressed as one moves away from the string, being proportional to $\exp[-\mathcal{H}_0 r]$ asymptotically. Since \mathcal{H}_0^{-1} will typically be of the order of the Compton wavelength of the fermion, we have a fermionic bound state wavefunction that has practically all its support concentrated at the core of the string.

IV. CONCLUSIONS AND FURTHER QUESTIONS

In this article we have studied some of the solutions to the spin-3/2 equation of motion in a cosmic string background. Our motivation was to look for physically interesting situations (for e.g. superconducting currents) in the context of supergravity theories with a gauged R -symmetry, where the gravitino ends up having a non-zero R -charge. We have also provided

an explicit construction of toy models of supergravity with gauged R -symmetry. The particle contents of the theories were chosen to cancel the $U(1)_R$ and mixed gravitational anomalies, thus leading to models where the symmetries are implemented in a consistent fashion. In the first example, both supergravity and R -symmetry were broken, while the second model had vacua where the supergravity was unbroken, leading to string configurations which were obtained explicitly. Such configurations could also arise in more realistic models, such as the extensions to MSSM considered in [9].

We obtained solutions to the classical field equations (Rarita-Schwinger equations) for the gravitino in the vortex background, in both the supergravity unbroken and broken phases. In both cases, the solutions possessed similarities with previously obtained solutions to the Dirac equation in vortex backgrounds, simply because they all turned out to be zero-modes of the transverse Dirac operator. In the SUGRA unbroken phase, the field configurations carried non-zero fermionic-currents along the string, pointing to the possibility of having superconducting states in the string. In the supergravity broken phase, however, we found solutions that did not carry any fermionic current or charge density, precluding the possibility of having persistent/superconducting R -currents, at least for the class of solutions found by us.

Our studies point to further interesting questions:

1. Is it possible to get solutions other than the ones already obtained, that will give rise to persistent currents? A closely related issue that would have to be understood is, how one would go about proving the existence of these superconducting currents. The answers are well-known for the spin-1/2 case, but the spin-3/2 case is intrinsically different and is yet to be understood.
2. There are other mathematically stimulating problems, namely index theorems for the Rarita-Schwinger equation in a string background.

We have concerned ourselves mostly with the interesting mathematical question of whether given gravitinos with non-zero R charges, they can give rise to trapped zero modes on R strings. However, there may be constraints on the existence of R strings in the universe depending on when they form. If the R -symmetry has to be broken at a scale greater than 10^{16} GeV, as may be required from considerations of naturalness [9,10], the energy density in strings may be too large to be accommodated by observations and the strings may have to be inflated away [11]. If use is to be made of gravitino zero modes, the symmetry breaking scale will have to be reduced *without* fine tuning the theory. It remains to be seen whether such models exist.

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